Hamiltonian description of neutrino oscillations

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Abstract

Neutrinos are the least understood among all known elementary particles. Their unique properties are the subject of current trends of experimental research.
During the presentation we shall briefly remind the standard approach to neutrino oscillations and show weak points of such reasoning.
We shall describe the innovative idea how to deal with this problem using generalized Hamiltonian description originally proposed by Gell-Mann and Goldberger in their famous work on The Formal Theory of Scattering [1].

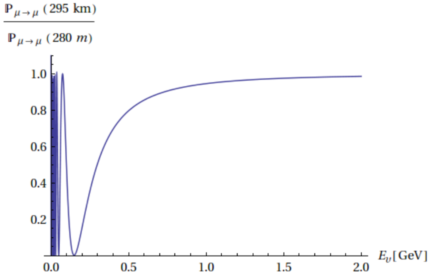
The short review [2] of the standard approach

$$l{=}e,\mu,\tau \qquad |\nu_l\rangle = \sum_i U_{li}|\nu_i\rangle \qquad i{=}1,2,3$$

$|\nu_i\rangle$ are plain waves with a specific momentum $\vec{q}$.

Probability in the standard approach

$$\mathbb{P}_{l\to l'}(t{=}L/c){=}|\sum_{i,j} U_{l'i}\mathrm{e}^{i\frac{m_i-m_j}{2|\vec{q}\,|}t}U^{\dagger}_{il}|^2$$



Drawbacks of the standard approach

Drawbacks by J.Rich [3]

- Why does the ν_e give a definite momentum rather than a definite energy?
- How can we change t into L/c when a neutrino with a definite momentum must have a wave function of infinite extent?
- Would not the time-of-flight and momentum measurements allow us to determine ν_i. When is know which ν_i was emitted and detected thus destory the interference that makes the oscillations?

Wave-packet treatments [4] eliminate some of these problems. Unfortunately, it is not clear what determines the size of the wave packet at the moment of its creation.

Old perturbation theory vs. Feynman

We will consider a system consisting initially of an unstable source particle and a target particle, both in a definite quantum state at some distance from each other.

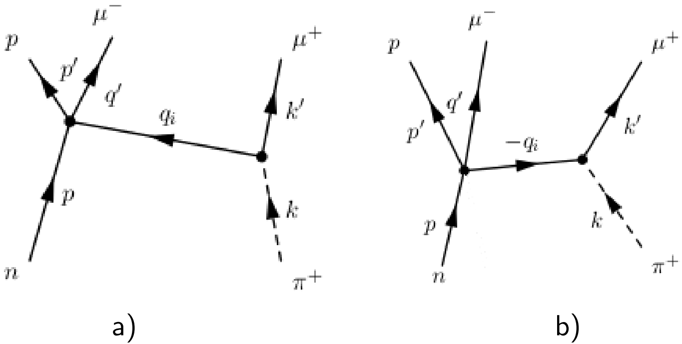


Figure: Two ordered diagram with a) neutrino b) antyneutrino as intermediate state

Wave package and amplitude

Initial state

$$|\phi_\mathrm{i}\rangle := \int \mathrm{d}^3 p \int \mathrm{d}^3 k\, \phi_n(\vec{p}, \vec{P}, \vec{L})\phi_\pi(\vec{k}, \vec{K})|\pi(\vec{k})n(\vec{p})\rangle$$

- pion π wave package $\phi_\pi(\vec{k},\vec{K}):=(\sqrt{2\pi}\sigma)^{-3}\exp\left\{-\frac{(\vec{k}-\vec{K})^2}{2\sigma^2}\right\}$
- neutron n $\phi_n(\vec{p},\vec{P},\vec{L}):=(\sqrt{2\pi}\sigma)^{-3}\exp\left\{-\frac{(\vec{p}-\vec{P})^2}{2\sigma^2}-i\vec{p}\vec{L}+iE_n(\vec{p})T\right\}$

Final state

$$|\phi_\mathrm{f}\rangle := |\mu^+(\vec{q}\,')\mu^-(\vec{k}\,')p(\vec{p}\,')\rangle$$

Second order amplitude

$$A(t) = \langle\phi_\mathrm{f}|\mathrm{e}^{-iH_I t}\frac{i\epsilon}{E_\mathrm{i} - H_0 + i\epsilon}H_I|\phi_\mathrm{i}\rangle$$

Assumptions and results

Since the wave-package treat uses the full Hamiltonian of QFT, it is necessary to introduce some simplifications in formula for amplitude, from which we derive $\mathbb{P}^{(\mathrm{GG})}_{\mu\to\mu}(t)$.

Assumptions

- $t \approx T$
- fermion factor depend only on mean value of momentum $\vec{K}$, $\vec{P}$, it is not necessary to integrate it over $\vec{k}$, $\vec{p}$
- intermediate neutrino have the energy $E_\nu \approx E_\pi - E_{\mu^+}$

Results

$$\frac{\mathbb{P}^{(\mathrm{GG})}_{\mu\to\mu}(t_1)}{\mathbb{P}^{(\mathrm{GG})}_{\mu\to\mu}(t_2)} = \frac{\mathbb{P}_{\mu\to\mu}(t_1)}{\mathbb{P}_{\mu\to\mu}(t_2)}$$

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