

Nieścisłe notatki - (ściśle na tablicy)

Rozwinięcie $1/N$ i model 't Hoofta.• Bibliografia:

1. "Large N" Amos Rudra
2. "Large N QCD" Aneesh Manohar
3. "Large N" Gerard 't Hooft
4. "Aspects of Symmetry" Sidney Coleman
5. "Non-Perturbative Field Theory" Frischman, Sonnenschein
6. G. 't Hooft, Nucl. Phys. B 72, 461 (1974)
7. G. 't Hooft, Nucl. Phys. B 75, 461 (1974)
8. Goto, Gross, Callan, Phys. Rev. D 13, 1694 (1976)
9. K. Meissner "Klasyczna Teoria Pola"
10. Wilson "Introduction to Graph Theory".

• Co to jest przybliżenie dużego N ?

Teoria z grupą cechowania $SU(N)$ to trudna teoria.

↓ $N \rightarrow \infty$

w pierwszym rzędzie rachunku zaburzeń odpadają wszystkie nieplanarne diagramy Feynmana

Podobno przychodzi się to przy jakościowej analizie QCD. ($SU(3)$):

- opis mezonów, (wzrost dwóch kwarków)
- opis barionów (N kwarków)

(tablica) trudniej

?

$$|N\rangle \otimes |N\rangle = N \otimes \bar{N} = \dots \oplus \dots \oplus \dots \oplus 0$$

?

$$\overbrace{|\mathbf{N}\rangle \otimes \dots \otimes |\mathbf{N}\rangle}^N = \overbrace{|\mathbf{N}\rangle \otimes \dots \otimes |\mathbf{N}\rangle}^N = \dots \otimes \dots \otimes \dots \otimes 0$$

• QCD • $S = -\frac{1}{4} \int d^4x F_{\mu\nu}^a F_a^{\mu\nu}$ $T^a \in su(N)$

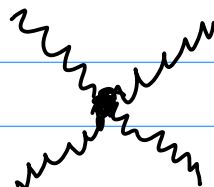
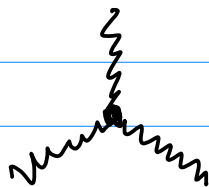
$$F_{\mu\nu} = F_{\mu\nu}^a T^a \Rightarrow F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + ig f^{abc} A_\mu^b A_\nu^c$$

• $S = \frac{1}{2g^2} \int F_{\mu\nu} F^{\mu\nu} d^4x$, $F_{\mu\nu} = ig F_{\mu\nu}^a T^a$, $A_\mu = ig A_\mu^a T^a$

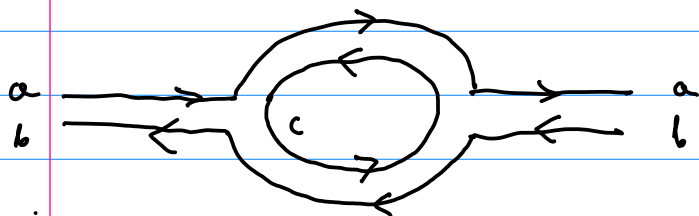
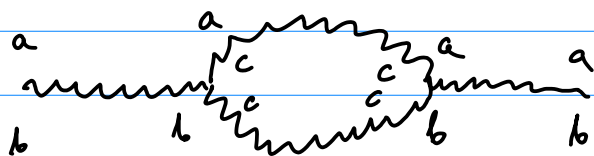
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]$$

• $S = -\frac{1}{g^2} \int \text{Tr}(F \wedge * F)$, $A = ig A_\mu dx^\mu$,
 $F = dA + A \wedge A$

↳ $\partial A \partial A$, $\partial A A A$, $A A A A$
 g , g , g^2



Na czym polega granica drzewo N ? $g\sqrt{N} = \lambda$
 $g \rightarrow 0, N \rightarrow \infty$
 $\lambda = \text{const}$



suma po c od 1 do N daje whiteł n_{gluon}
 $\sim g^2 N = \lambda$

$$g^4 N^2 = \lambda^2$$

$$g^6 N = \frac{\lambda}{N^2}$$

- Jak to sformalizować?

przy czym $[F_{\mu\nu}]^a_b = \partial_\mu [A_\nu]^a_b - \partial_\nu [A_\mu]^a_b + i \frac{g}{N} [A_\mu A_\nu]^a_b$

$$S = -\frac{1}{4} \int \text{Tr}(F_{\mu\nu} F^{\mu\nu}) d^4x + \int d^4x \bar{\Psi} (i\not{D} - m) \Psi$$

$$\Psi = \begin{pmatrix} \psi_1 \\ \vdots \\ \psi_n \end{pmatrix}, \quad \bar{\Psi} = (\bar{\psi}_1, \dots, \bar{\psi}_n)$$

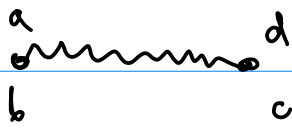
$$\not{D} = \not{D}_\mu \gamma^\mu = \partial_\mu \gamma^\mu + i \frac{g}{N} A_\mu \gamma^\mu$$

$$\langle 0 | T(\psi^a(x) \bar{\psi}^b(y)) | 0 \rangle = \delta^{ab} S_F(x-y)$$

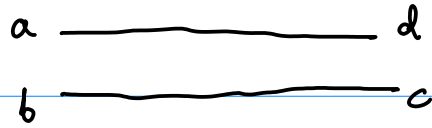
$$\langle 0 | T([A_\mu]^a_b [A_\nu]^c_d) | 0 \rangle = D_{\mu\nu}(x-y) \cdot \mathbb{I}$$

$$\mathbb{I} = \frac{1}{2} \left(\delta^a_d \delta^c_b - \frac{1}{N} \delta^a_b \delta^c_d \right)$$

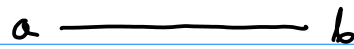
$$(T_i)^a_b (T_i)^c_d = \mathbb{I}$$



\longleftrightarrow



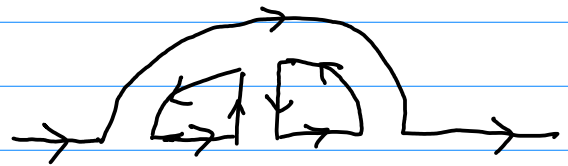
\longleftrightarrow



\longleftrightarrow



\longleftrightarrow



Jak udowodnić, że tylko diagramy planarne mają wkład do amplitud?

predefiniujemy pole : $\Psi \rightarrow \sqrt{N} \Psi / g$

$A \rightarrow \frac{\sqrt{N}}{g} A$

$$\Psi \Psi A \frac{g}{\sqrt{N}} \rightarrow \Psi \Psi A N / g^2$$

$$A A \rightarrow A A N \frac{1}{g^2}$$

$$\Psi \Psi \rightarrow \Psi \Psi N / g^2$$

$$\partial A A A \frac{g}{\sqrt{N}} \rightarrow \partial A A A N \frac{1}{g^2}$$

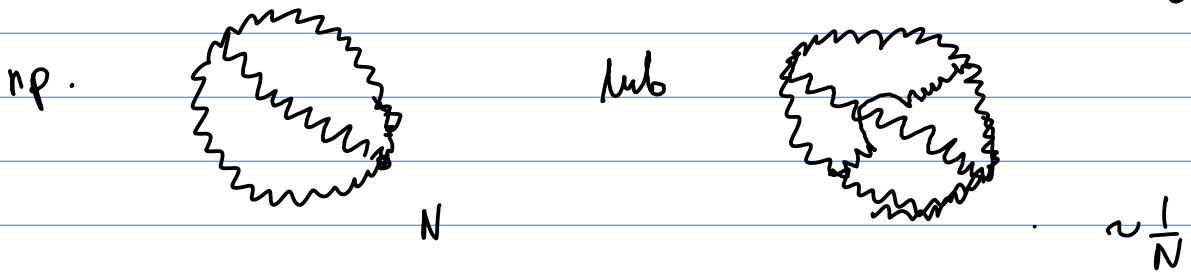
$$A A A A \frac{g^2}{N} \rightarrow A A A A N \frac{1}{g^2}$$

$$S \rightarrow \frac{N}{g^2} (FF + \bar{\Psi} (D - m) \Psi)$$

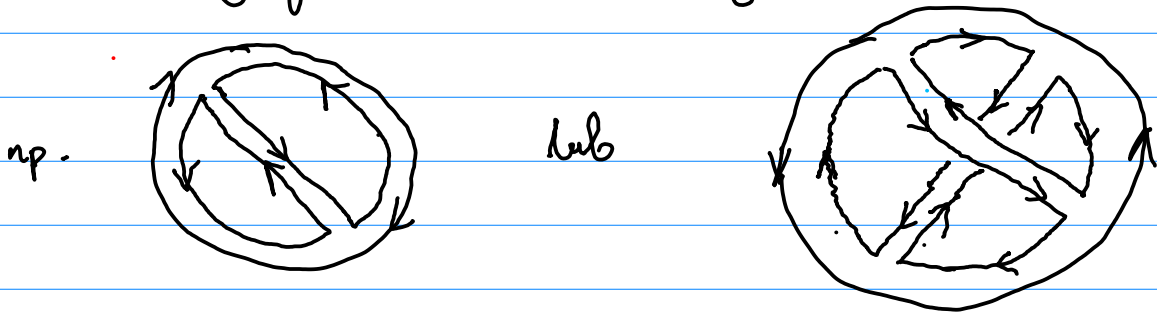
propagator $\sim \frac{1}{N}$, wierzchołek $\sim N$, petla $\sim N$

Procedura wielokątowania diagramów z gluonami:

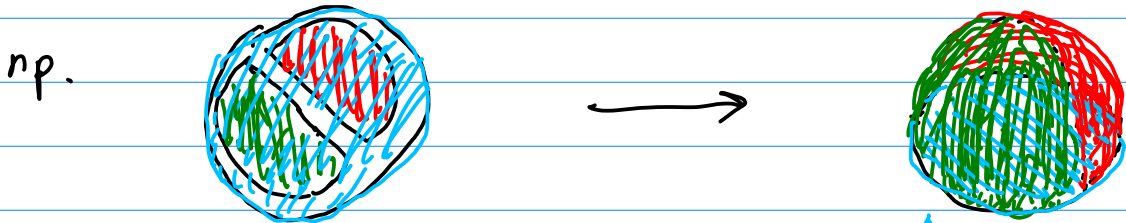
1. Weź dowolny diagram próżniowy (spójny)



2. Narysuj go w notacji 't' Kobałki:



3. Do każdej pętli indeksowanej przypisz ściankę:

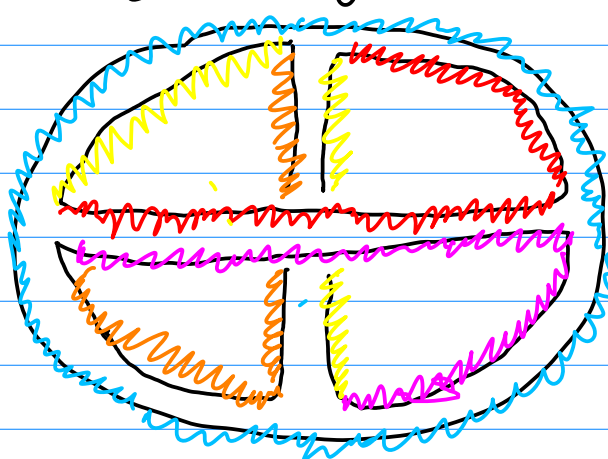


$$N N^2 N^2 \frac{1}{N^3} \sim N^2$$

pod spodem jest niebieska ścianka

Tredny obrazek:

zlepiany ścianki o tym samym kolorem ściankami, które są fragmentami jednej ścianki.



$$\sim N^4 \frac{1}{N^6} N^2 = 1$$


Terzowa rewolucja!

(dwie dziury)

Bernadrije ne te nyankei.

Na tablicy będą lepsze.



kolory  są tożsame. Używam różnych żeby było prościej zobaczyć o co chodzi.

Które figury dają istotny wkład do amplitud?

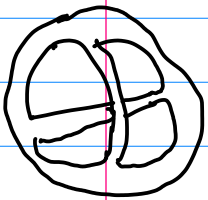
Niech $j =$ liczba wierzchołków + liczba pętli indeksowych

- liczba propagatorów =

= $l.$ wierzchołków + $l.$ ścian - $l.$ krawędzi

(= 2 dla grafów płaskich)

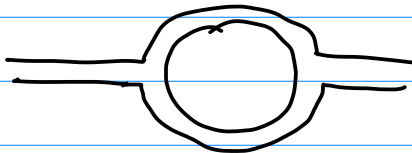
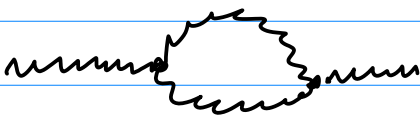
= $2 - 2H - B$ gdzie H to liczba handle, a B to liczba dziur.



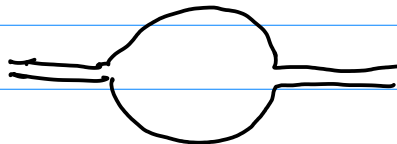
$$N^2 \frac{1}{N^6} N^4$$

• G z kwarkami?

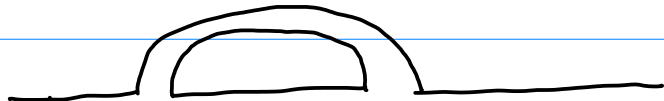
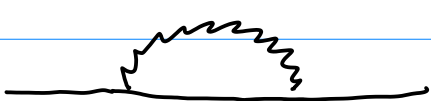
Owóid na tablicy.



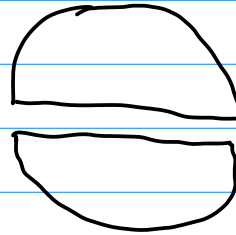
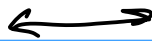
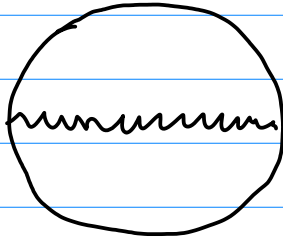
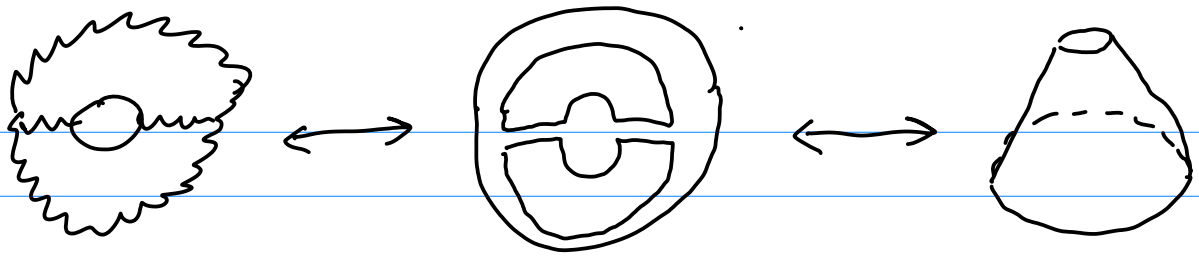
j



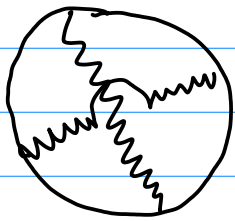
$j-1$



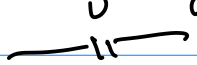
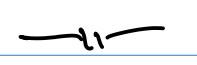
Gryfi wewnętrzne pętle kwarkowe dają dziury w diagramach przeniowych.



$$\sim N^2 \frac{1}{N^3} N^2 = N$$



$$\sim N^4 \frac{1}{N^6} N \sim \frac{1}{N}$$

1. Diagramy próżniowe gluonowe $\sim N^2$
2.   kwarkowe $\sim N$ (planarne)

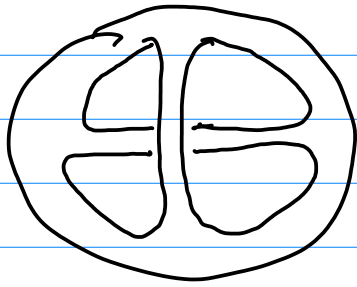
Co 2 innymi diagramami?

$$\begin{aligned} \langle G_1 G_2 \rangle &\sim N^0 & M &= \bar{\Psi} \Psi \text{ - mezon operator} \\ \langle G_1 \dots G_n \rangle &\sim N^{2-n} & & \text{(czy to dobre} \\ \langle M_1 M_2 \rangle &\sim N^{-1} & & \text{wartość?) } \\ \langle \sqrt{N} M_1 \sqrt{N} M_n \rangle &\sim N^{1-\frac{n}{2}} \\ \langle G_1 \dots G_n \sqrt{N} M_1 \dots \sqrt{N} M_m \rangle &\sim N^{1-n-\frac{m}{2}} \end{aligned}$$

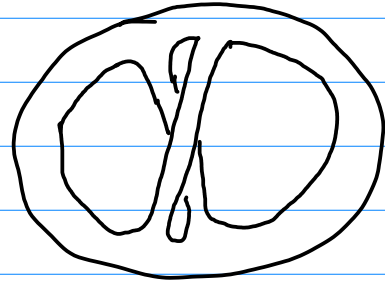
$$\langle O_1 \dots O_n \rangle = \frac{1}{iN} \frac{\partial}{\partial z_1} \dots \frac{1}{iN} \frac{\partial}{\partial z_n} W(z)$$

$$W(z) = \int D\psi DA e^{iS(\psi, A) + \bar{z}} \quad \bar{z} = \int d^4x \sum_i N \bar{z}_i O_i$$

Dodatkowy rysunek:

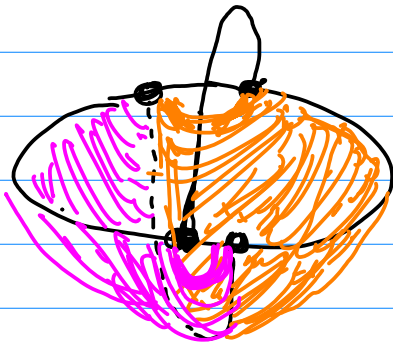


przesuwamy
beleczki →



→ naciągamy

beleczki →



dopiero teraz potoczemy odp. →

kręćdzie ścianami, ale na
kole podzielimy ją na 4 części

→

Chyba jednak

